The Detection Statistics of Neutrons and Photons Emitted from a Fissile Sample

Andreas Enqvist[a,*], Imre Pázsit[a], Sara A. Pozzi[b]

[a] *Department of Nuclear Engineering, Chalmers University of Technology SE-412 96 Göteborg, Sweden*
[b] *Department of Nuclear Engineering and Radiological Sciences, University of Michigan, Ann Arbor, MI 48109, USA*

Abstract

The purpose of this paper is to present new analytical derivations to describe the emission and detection statistics of neutrons and photons generated in and emitted from fissile samples, with absorption included. The results of the analytical approach are compared with and validated by corresponding Monte Carlo simulations. The joint statistics of the generated and detected neutrons and photons is also described. The analytical model described in this paper accounts for absorption and detection, thus extending the model presented in previous studies. By using this new, improved model, one can investigate the relative feasibilities of measuring neutrons, gamma photons or combinations thereof, for the analysis of a specific fissile sample. In fact, we show that for larger mass samples photon absorption in the sample strongly decreases the multiplicity of emitted photons, whereas this is not the case for neutrons. The results suggest that although photons have a larger initial (source) multiplication, neutrons might be more favourable to measure in the case of large samples because of the increasing self-shielding effect for gamma photons.

Key words: nuclear safeguards, neutron and photon numbers, number distributions, generating functions, master equations, multiplicity, joint distribution.

* Corresponding author

Email addresses: andreas@nephy.chalmers.se (Andreas Enqvist), imre@nephy.chalmers.se (Imre Pázsit), pozzisa@umich.edu (Sara A. Pozzi)

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1. Introduction

In non-destructive assay of nuclear materials, the statistics of the number distribution of neutrons and gamma rays emitted by fissile samples is of high importance. The multiplicities of neutrons [1] and photons [2] generated in fissile samples with internal multiplication have been investigated in the past. Investigations of the effect of including absorption in the model have also recently been made. The implicit master equations are the starting point for the generating functions of the neutron and photon number distribution. These equations have also been used in the past for calculating factorial moments. For practical reasons, factorial moments are usually only interesting up to the third or fourth order. Using multiplicity and coincidence measurements, one can deduce the sample mass and isotopic composition of a certain sample.

A characteristics of the factorial moments is that, experimentally, they are difficult to measure beyond the third moment (triples), whereas from an analytical or calculational point of view, the high order moments beyond triples become increasingly long and tedious to derive by hand. The present work describes a development in the latter area where calculations of the number distribution of fissile samples also yields results for the factorial moments.

In contrast to the factorial moments, the probabilities $P(n)$ and $F(n)$ of emitting n neutrons or gamma photons respectively, are interesting up to large values of n. The necessary number of terms of $P(n)$ or $F(n)$ that need to be calculated is determined from the condition that the cumulative probability should be sufficiently close to unity. In some cases the vale of n can exceed 50 for both neutrons and photons.

Including neutron and photon absorption in the model affects the statistics of the number distribution in several ways. For neutrons, the process or radiative absorption eliminate them from the fission chain and causes the observable (leaked) neutrons to reduce in number. For photons the dependence is more involved, because both the absorbtion of neutrons and the photons themselves will affect the photon distribution. All of these effects will vary with the main parameter of the sample being investigated, i.e. the sample mass. An increase in mass affects the probability of both induced fission, as well as absorption of both neutrons and gamma photons for large samples in the few kg mass range.

The process of detection allows us to observe the statistics of emission from a given sample. There is always a certain detector efficiency involved in the process of detection. One way of modelling this effect was investigated in [3], in which the detectors are assumed to be embedded into the sample. This model is not realistic since the detector contributes to the absorption inside the sample. In particular, a 100% detector efficiency means that the detector material completely suppresses that of the sample and hence the measurement yields information on the detector properties only (pure absorption without multiplication). In this work we will present a different way of accounting for absorption and detection, with the latter only pertaining to the particles that leaked out from the sample. In this way the detection process can be described more realistically. The results of the analytical calculations are compared to those from Monte Carlo simulations.

2. Theory

The master equations, or Chapman-Kolmogorov equations, for the generating functions of the number of neutrons and photons in a sample with both spontaneous and induced fission have been derived in references [1, 2]. In both cases, it is assumed in these calculations that the probability of a first collision before escape of the sample of an arbitrary neutron, p, is known. That model is here expanded to account for absorption and detection, and the probabilities for these events are included explicitly in the equations. With these extensions, the equations give the detection statistics in an analytical way, and can be used to plan experiments, or as an indicator when comparing to numerical results usually obtained using Monte Carlo codes.

2.1. Neutron distributions

In earlier models of the statistics of neutron and photon emission from a fissile sample[1, 2, 4], the absorption of neutrons and gamma photons was not accounted for. It was assumed that each neutron has a probability p of inducing fission, or failing to do so and hence escaping with a probability $1-p$.

The probability generating functions (PGFs) $h(z)$ and $H(z)$ of $p_1(n)$ and $P(n)$ describing the number of neutrons generated by one initial neutron or one initial neutron event (spontaneous fission), are defined as

$$h(z) = \sum_n p_1(n) z^n \quad \text{and} \quad H(z) = \sum_n P(n) z^n, \tag{1}$$

respectively. These are used in the coupled backward master equations for neutrons [1]:

$$h(z) = (1-p)z + pq_i[h(z)] \tag{2}$$

and

$$H(z) = q_s[h(z)]. \tag{3}$$

Here,

$$q_s(z) = \sum_n p_s(n)z^n \quad \text{and} \quad q_i(z) = \sum_n p_i(n)z^n \tag{4}$$

stand for the generating functions of the number of neutrons generated in a spontaneous or an induced fission, respectively. For finding the statistics of the particles one needs to observe that $p_1(n)$ and $P(n)$ are the Taylor expansion coefficients of $h(z)$ and $H(z)$, respectively [4]:

$$p_1(n) = \frac{1}{n!}\frac{d^n h(z)}{dz^n}\bigg|_{z=0} \quad \text{and} \quad P(n) = \frac{1}{n!}\frac{d^n H(z)}{dz^n}\bigg|_{z=0}. \tag{5}$$

As can be noted, the expressions are evaluated at $z = 0$, in contrast to taking them at $z = 1$, which is the case when searching for the factorial moments of the neutrons generated in spontaneous or induced fission ν_s, ν_f. In order to have compact expressions, modified moments were introduced in [4], which differ in value from the traditional nuclear factorial moments, defined as:

$$\frac{d^n q_\alpha(h)}{dh^n}\bigg|_{z=0} = q_\alpha^{(n)}(h)\big|_{z=0} = q_\alpha^{(n)}[p_1(0)] = \overline{\nu}_{\alpha n} \quad ; \quad \alpha = s, i, \tag{6}$$

where $p_1(0)$ is the probability of having zero neutrons generated when starting with one initial neutron. This initial probability needs to be found since higher order terms of $p_1(n)$ and $P(n)$ all depend on it, as well as it appears also in the modified factorial moments. Note can be made of the fact that this probability is highly dependent on the sample mass, and whether or not absorption and detection are included into the model. One expects this probability to increase with the inclusion of absorption and detection probability. Absorption can remove the initial neutron or all neutrons generated in the short chains started by the first neutron, likewise, one can end up with zero neutrons detected even if the single neutron started a long chain of fissions, if the detection probability is low.

In the neutron probability balance equation, the event of absorption can be included into the fission distribution, because when looking at the progeny of neutrons it is the same as a fission event with zero neutrons generated. Therefore one can include the absorption by a suitable increase of the first collision probability from a value p to p'. The new first collision probability then accounts for both fission and absorption, and the "reverse" probability $1-p'$ now properly describes the probability for a neutron to escape the sample and become available for external detectors to register. To maintain normalization, the probabilities $p_i(n)$ for $n > 0$ will need to be decreased according to the following formula:

$$\widetilde{p}_i(n) = \frac{p'-p}{p'}\delta_{n,0} + \frac{p}{p'}p_i(n). \tag{7}$$

The generating function of $\widetilde{p}_i(n)$, Eq (4), will also change accordingly into $\widetilde{q}_i(z)$. The first master equation (2) will now read:

$$h(z) = (1-p')z + p'\widetilde{q}_i[h(z)], \tag{8}$$

and describes the leaked out neutrons. In the numerical work, the values of p, p' and the probabilities $p_i(n)$ are taken from the code MCNP-PoliMi [5], which contains extensive nuclear data tables. This will have the advantage that in the comparisons with Monte Carlo simulations, it is assured that the same nuclear data are used in both the simulations and the analytical calculations.

The process of detection can be added in a straightforward manner by considering only the neutrons that have already leaked out of the sample, since those are the ones available for detection by external detectors. Using a detection probability for neutrons, ϵ, we can create a generating function $\varepsilon(z)$ of the binary probability distribution (i.e. either zero or one neutron) of the number of neutrons detected per neutron emitted from the sample:

$$\varepsilon(z) = \epsilon z + (1-\epsilon). \tag{9}$$

The new generating functions that also include the detection process are given by:

$$h_d(z) = h[\varepsilon(z)] \quad , \quad H_d(z) = H[\varepsilon(z)]. \tag{10}$$

The derivatives needed for finding the factorial moments and the number distribution change in a simple way

$$\frac{d^n h_d(z)}{dz^n} = \frac{d^n h(z)}{dz^n}\cdot(\epsilon)^n \quad , \quad \frac{d^n H_d(z)}{dz^n} = \frac{d^n H(z)}{dz^n}\cdot(\epsilon)^n. \tag{11}$$

For factorial moments the full change is

$$\widetilde{\nu}_{d,n} = (\epsilon)^n \cdot \widetilde{\nu}_n. \tag{12}$$

This is due to the factorial moments (multiplicites) being evaluated at $z = 1$. For the number distribution on the other hand, the evaluation at $z = 0$ resulted in modified moments which depend on the probability $p_1(0)$. This probability changes when absorption and detection in accounted for, so even if the derivatives change formally in the same manner as for the factorial moments, the modified moments will also obtain new numerical values.

In our previous work [4], i.e. with no absorption and detection, the modified moments were given by

$$\left.\frac{d^n \widetilde{q}_\alpha(h)}{dh^n}\right|_{z=0} = \widetilde{q}_\alpha^{(n)}(h)\big|_{z=0} = \widetilde{q}_\alpha^{(n)}[p_1(0)] = \overline{\nu}_{\alpha n} \quad ; \quad \alpha = s, i. \tag{13}$$

In the case of detection, $p_d(0)$ will replace $p_1(0)$, which is solved from the N-th order polynomial

$$p_d(0) = (1-p')(1-\epsilon)+p\,\widetilde{q}_i\,[p_d(0)] = (1-p')(1-\epsilon)+p'\sum_{n=0}^{N}\widetilde{p}_i(n)[p_d(0)]^n. \tag{14}$$

where N is the largest neutron multiplicity in an induced fission, which is set to $N = 8$ in the case of plutonium.

Using these properties we can now derive the detection statistics from the Taylor expansion:

$$p_d(n) = \frac{1}{n!}\left.\frac{d^n h_d(z)}{dz^n}\right|_{z=0} \quad \text{and} \quad P_d(n) = \frac{1}{n!}\left.\frac{d^n H_d(z)}{dz^n}\right|_{z=0}. \tag{15}$$

The terms in the probability distribution can now be calculated recursively because the starting master equation is in implicit form. This fact means that the probabilities $P(n)$ will occur in the expressions for $P(m)$, where $m > n$. This makes it computationally favourable to use the symbolic computation code Mathematica [6], which was used to derive the higher order terms.

The analytic model allows us to readily analyze the dependence of the measured quantities on simple parameters: the probability to induce fission, which is a parameter that increases with mass, and can be calculated from the mass of the sample, provided the density is known; the absorption probability which also depends on sample size and composition; and finally the detection efficiency, which can be changed to reflect what type of detector is used, such as fast scintillation detectors or large arrays of helium tubes in form of multiplicity counters.

2.2. Photon distributions

As in the case of neutrons, a set of coupled backwards master equations have earlier been derived and used to find the statistics of the generated photons [2, 4]. The starting equations were:

$$g(z) = (1-p) + p r_i(z) q_i[g(z)] \tag{16}$$

and

$$G(z) = r_s(z) q_s[g(z)], \tag{17}$$

where $g(z)$ and $G(z)$ are the probability generating functions of $f_1(n)$ and $F(n)$, describing the number of generated photons when starting with one neutron or one source event, respectively:

$$g(z) = \sum_n f_1(n) z^n \quad , \quad G(z) = \sum_n F(n) z^n. \tag{18}$$

One needs also to use the nuclear data for the distribution of photons generated in one induced or spontaneous fission respectively, defined as:

$$r_i(z) = \sum_n f_i(n) z^n \quad , \quad r_s(z) = \sum_n f_s(n) z^n. \tag{19}$$

When performing the differentiations to find the probability distributions $f_1(n)$ and $F(n)$, one will, just as in the case of neutrons, encounter modified moments defined as:

$$\left.\frac{d^n r_\alpha(z)}{dz^n}\right|_{z=0} = n!\, f_\alpha(n) \equiv \overline{\mu}_{\alpha n}; \qquad \alpha = s, i. \tag{20}$$

In the calculation of these modified moments, which refer to that of the photons, also the modified moments of neutrons will appear, due to the structure of the equations (16) and (17). Physically, this is due to the fact that the internal multiplication of photons is only due to the neutron branching, hence the corresponding neutron moments will also appear. However, these neutron moments will not be the same as the ones that are derived for the "pure" neutron distribution, equations (2) and (3). The reason is that for the calculation of these modified neutron moments that appear in the gamma photon distributions, the factor $p_1(0)$ in Eq. (6) will be replaced by the factor $f_1(0)$ in the case of photons in the corresponding expressions.

To further extend this model to account for detection statistics instead of merely the number of generated particles [4], one needs to include both absorption and the process of detection, which takes place with a certain probability, referred to often as the detection efficiency, ϵ.

In the case of neutrons, the absorption was taken into account by modifying the probability distribution $\widetilde{p}_i(n)$ and changing the first collision probability from p to p'. For photons the situation will be slightly different, because leakage and absorption of a neutron will both lead to zero generated photons. Hence the parameter p in (16) remains that of the probability to induce a fission. The generating function $\widetilde{q}_i(z)$, will still be used, since with zero neutrons generated, the branching process will stop, and no more photons can be generated.

Gamma absorption will be accounted for by the probability l_γ that describes the leakage probability for one single photon, likewise $(1 - l_\gamma)$ is the probability for a created photon to be absorbed and not escape the sample. The gamma capture will be accounted for by an additional generating function $\ell_\gamma(z)$,

$$\ell_\gamma(z) = l_\gamma z + (1 - l_\gamma). \tag{21}$$

Here $\ell_\gamma(z)$ is the generating function of the binary probability distribution of gamma photons leaving the sample per initial photon. Due to the simple form of this relationship, the factorial moments of the leaked out neutrons are simply the factorial moments for the generated photons times a leakage factor:

$$\widetilde{\mu}_{l,n} = (l_\gamma)^n \cdot \widetilde{\mu}_n. \tag{22}$$

The master equations for the leaked out photons are then given as:

$$g_l(z) = g[\ell_\gamma(z)], \qquad G_l(z) = G[\ell_\gamma(z)]. \tag{23}$$

To separate the statistics of the photons that undergo detection compared to the larger numbers of photons that escape the sample, an extra equation is added that describes the probability for one photon to undergo detection or not. The detection efficiency, ϵ_γ, is used:

$$\varepsilon_\gamma(z) = \epsilon_\gamma z + (1 - \epsilon_\gamma). \tag{24}$$

The probability distribution can now be extracted by using the master equations

$$g_d(z) = g\big[\ell_\gamma\{\varepsilon_\gamma(z)\}\big] = (1-p) + p r_i(z)\widetilde{q}_i[g_d(z)] \tag{25}$$

and

$$G_d(z) = G\big[\ell_\gamma\{\varepsilon_\gamma(z)\}\big] = r_s(z) q_s[g_d(z)]. \tag{26}$$

Since one evaluates the expressions at $z = 0$ to get the probability distribution, modified moments are created as earlier. In the case of detected photons these modified moments will depend on $f_d(0)$ instead on $f_1(0)$, i.e. the probability of having zero neutrons *detected* when starting with one initial neutron in the sample. This quantity can be found by setting $z = 0$ in Eq. (25), and finding the root of the finite degree polynomial that arises:

$$\begin{aligned} f_d(0) &= (1-p) + p\, r_f[\ell_\gamma\{\varepsilon_\gamma(0)\}]\, q_f[f_d(0)] = \\ &= (1-p) + p\left(\sum_{n=0}^{\sim 24} f_f(n)[\ell_\gamma\{\varepsilon_\gamma(0)\}]^n\right)\cdot\sum_{n=0}^{8} p_f(n)[f_d(0)]^n. \end{aligned} \tag{27}$$

The modified moments using this new initial term are defined as follows when we include absorption and detection into the model:

$$\left.\frac{d^n r_\alpha[\ell_\gamma\{\varepsilon_\gamma(z)\}]}{dz^n}\right|_{z=0} \equiv \overline{\mu}_{d,\alpha n}; \qquad \alpha = s, i, \tag{28}$$

$$\left.\frac{d^n q_\alpha(g_d)}{dg_d^n}\right|_{z=0} \equiv \overline{\nu}_{d,\alpha n}; \qquad \alpha = s, i. \tag{29}$$

The factors are straightforward to calculate, but lead to expressions that contain sums that have an increasing number of terms for higher order moments. The formulae obtained for the number distributions, both in the case of neutrons and photons, contain several quantities that are based on nuclear physics constants (fission neutron and gamma photon multiplicities), weighted by factors depending on the first collision probability, absorption probabilities etc. Among those quantities one also finds the modified moments of $\overline{\nu}_{\alpha n}$ and $\overline{\mu}_{\alpha n}$. When calculating higher order terms and getting longer expressions one notes that $\overline{\nu}_{\alpha n}$ becomes zero for $n > 8$, due to the fact that fission multiplicities are zero for so large numbers. In the same manner $\overline{\mu}_{\alpha n}$ vanishes for a larger n. Considerations like these make the otherwise rapidly growing expressions more manageable, however the use of computer software to handle the numerical evaluations remains a necessity.

A further fact to be mentioned concerns the formal equivalence between the probabilities $P(n)$ and the corresponding n-th order factorial moments of the neutrons and photons generated in the sample, as noted in Ref. [4]. In [4] it was noted that the factorial moments of the number of neutrons and photons generated in the sample could be obtained from the probabilities by replacing the modified moments with the ordinary moments which are based on the pure nuclear data. It is seen from the expressions derived in this paper that the same formal equivalence exists between the probabilities $P_d(n)$ of the detected neutrons and photons and the corresponding factorial moments. Thus by finding the probability distribution of the detected particles, one receives the multiplicities as a limiting simplified case up to the same high order as the distribution was determined. This order is generally much higher than the third or fourth, up to which the factorial moments are usually computed.

2.3. Joint distributions

In a more complete description of emission of neutrons and gamma photons from fissile samples, one can extend the description to the joint statistics of neutrons and photons. For this one also would need the joint number distributions $p_s(n,m)$ and $p_i(n,m)$ of n neutrons and m photons emitted in a spontaneous and an induced fission, respectively. Unfortunately there are no data for such joint distributions in the literature. Generally, the generation of neutrons and photons is considered to be independent, hence the above distributions are just products of the individual distributions. The corresponding PGF's are defined as

$$q_s(z,y) = \sum_n \sum_m p_s(n,m) z^n y^m \quad , \quad q_i(z,y) = \sum_n \sum_m p_i(n,m) z^n y^m. \quad (30)$$

If $b_1(n,m)$ is the probability to obtain n neutrons and m photons generated by one initial neutron, a backward-type master equation can be readily derived by considering the two mutually exclusive events of not having or having a first collision before leaking out:

$$b_1(n,m) = (1-p)\, \delta_{n,1}\, \delta_{m,0} +$$

$$p\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}p_i(k,l)\sum_{\substack{n_1+\cdots+n_k=n\\ m_1+\cdots+m_k=m-l}}\prod_{i=1}^{k}b_1(n_i,m_i). \tag{31}$$

Defining the generating function

$$c(z,y)=\sum_n\sum_m b_1(n,m)z^n y^m, \tag{32}$$

from (31) one obtains

$$c(z,y)=(1-p)z+pq_i[c(z,y),y]. \tag{33}$$

In the same manner an equation for the probability generating function $B(n,m)$ of the probability to have n neutrons and m photons generated when starting with one source event, can be derived as:

$$C(z,y)=q_s[c(z,y),y]. \tag{34}$$

Also for the joint distributions, from the practical point of view it is more interesting to determine the distribution of detected particles. Using the same concepts as before, one defines the generating functions of the leakage probability of a single neutron and photon, respectively:

$$\begin{cases} \ell_n(z)=l_n\,z+(1-l_n),\\ \ell_\gamma(y)=l_\gamma\,z+(1-l_\gamma), \end{cases} \tag{35}$$

where l_n and l_γ are the leakage probabilities of neutrons and photons, respectively. Actually, in the earlier notations, l_n is simply equal to $1-p'$. The values of these parameters will naturally vary with the main parameter of the investigated sample, which is the sample mass. Detection can then be included in the same manner by using the earlier defined generating functions

$$\begin{cases} \varepsilon_n(z)=\epsilon_n\,z+(1-\epsilon_n),\\ \varepsilon_\gamma(y)=\epsilon_\gamma\,z+(1-\epsilon_\gamma), \end{cases} \tag{36}$$

with ϵ_n and ϵ_γ being the detection efficiencies of neutrons and photons respectively. The coupled master equations for the detection statistics are then given as:

$$\begin{aligned} c_d(z,y) &= (1-p)\,\ell_n\{\varepsilon_n(z)\}+pq_i[c_d(z,y),\ell_\gamma\{\varepsilon_\gamma(y)\}],\\ C_d(z,y) &= q_s[c_d(z,y),\ell_\gamma\{\varepsilon_\gamma(y)\}]. \end{aligned} \tag{37}$$

The individual distributions and corresponding defining equations for the neutrons or photons can now be found as special cases: $y = 1$ in the Eqs. (37) gives the neutron distributions equivalent of Eqs. (10), while setting $z = 1$ in the Eqs. (37) gives Eqs. (25), (26). The sought joint probability distributions are calculated as the n, m-th derivatives of $c_d(z, y)$ and $C_d(z, y)$ with regard to z and y respectively.

3. Results

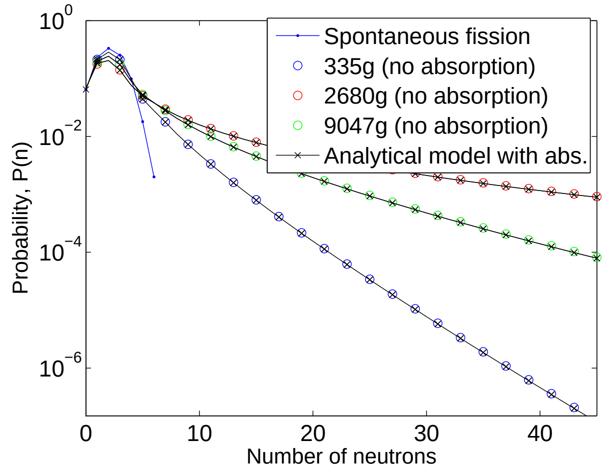
The number distributions were calculated for neutrons and photons separately, as well as for joint distributions, for plutonium metal spheres of varying mass. The values obtained were compared to simulations with the code MCNP-PoliMi [2, 5, 7]. To this end, MCNP-PoliMi had to be modified to supply the necessary tallies. The values of the collision probability p, and that of the nuclear physics constants such as the fission parameters $p_s(n)$, $f_i(n)$, etc. were taken from MCNP-PoliMi runs for the analytical model. The probability to induce fission varies with the mass of the sample. The values of p are shown in Table 1 below for the three samples for which we have performed calculations.

Sample	Mass (kg)	p
1	0.335	0.0852
2	2.680	0.1678
3	9.047	0.2461

Table 1: **Probability to induce fission, p, for one neutron depending on the mass of the sample. The metal samples have a composition of 80 wt% Pu-239 and 20 wt-% Pu-240, and a density of 15.9 g/cc.**

The analytical model in the case of generated particles (no absorption included), has earlier been validated against Monte Carlo simulations [4] with very good agreement. In MCNP-PoliMi a spherical encompassing idealized detector was used to measure all outcoming particles without reflecting them back to the sample to change the statistics. Thus the result obtained was the statistics of the number of emitted particles.

As can be seen in Fig. 1, for photons there is a very evident effect of self-shielding. The probabilities of high numbers of photons escaping the sample are reduced significantly when absorption is taken into account. The results show that for realistic samples, which might be investigated with typical



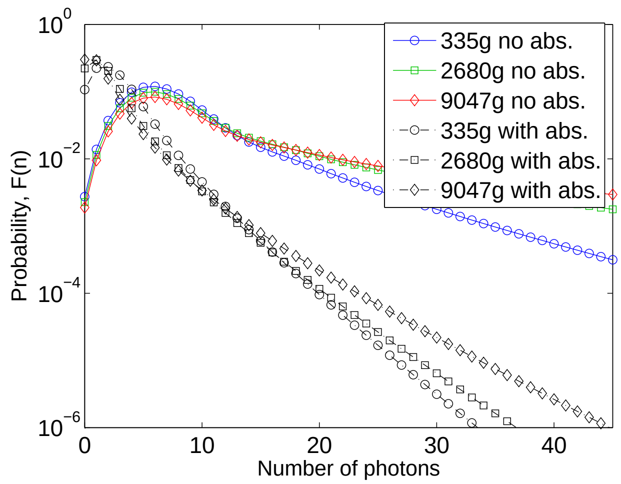


Figure 1: Comparison of the number distribution for neutrons and photons, with and without accounting for absorption. Colored markers are without absorption and black lines with absorption included. For neutrons the lines coincide, while for photons a major change is seen.

non-destructive assay (NDA) techniques, the initial advantage of having high photon multiplicities diminishes, because the internal absorption of photons is much greater than that for neutrons in materials of high atomic numbers.

Figure 2 shows the good agreement between the analytical model and the numerical simulations when looking at the emitted particles. Photon

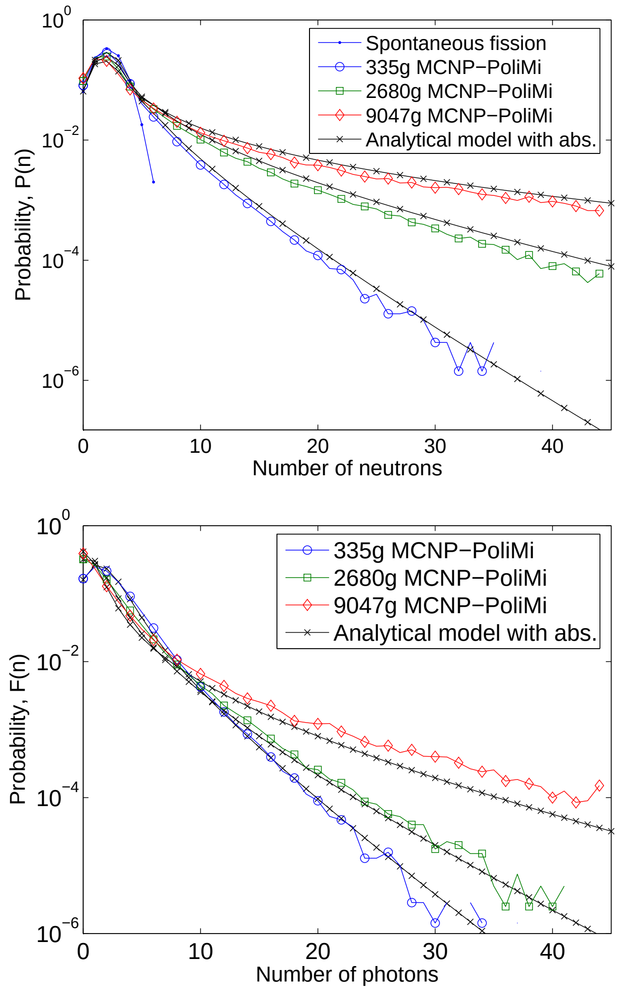


Figure 2: Comparison between numerical simulations and analytical results where the effect of absorption is accounted for. As can be seen, the agreement is good in general. At low probabilities, i.e. with higher numbers of particles, the Monte Carlo data are difficult to compare to, due to poor statistics caused by the finite number of histories run.

detectors can still be very useful if we consider a scenario where the sample is heavily screened with low Z materials, and/or materials with high neutron absorption cross sections, which do not screen gamma photons. In such cases photon detectors can be advantageous compared to neutron detectors. Different types of shielding can be accounted for in the model by correctly changing the detection efficiency or alternatively increasing the absorption probability which might be easier in the case of photons.

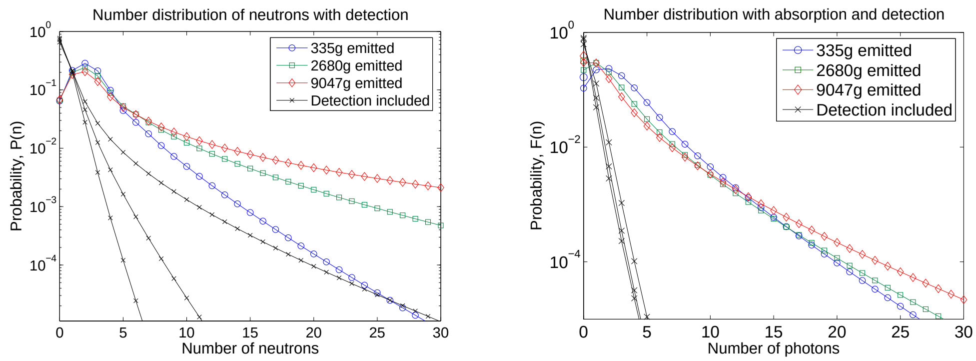


Figure 3: Probability distributions with detection included. The shape of the curves will be dependent on the detector efficiency ϵ_x, which was taken as 10% for neutrons and 20% for photons in these plots.

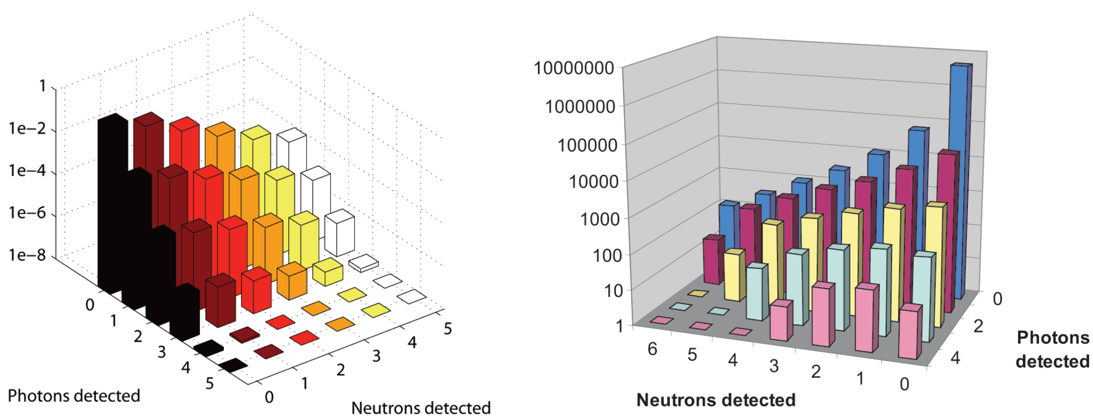


Figure 4: The left hand side graph shows data from the analytical model where we have assumed a larger detection efficiency for neutrons compared to photons, e.g. a sample shielded with a material of high atomic mass. The right hand side graph is from MCNP-PoliMi, showing the detection statistics for a detector setup using six scintillator detectors.

In the case of gamma photons one can note the somewhat unexpected result that it is more likely to detect relatively high multiplicities, such as triples and quadruples, for a lighter sample than for a heavy one. The reason is that, although the total amount of generated photons is higher for the heavier samples, the self-shielding effect counteracts this. The fast growing self-shielding (with increasing mass) constitutes of course a disadvantage. Note that these probabilities are per source event, and in a sample of higher mass the number of spontaneous fissions will be higher.

The effect of the detection process in the model, so that the distribution shows the number of detected particles rather than the emitted particles, is shown in Fig. 3. The change of the distributions is directly linked to the detection efficiency, and the lower the detection efficiency, the more difficult it is to see higher order multiplicities which is visible in the figures which compare with the distribution of emitted particles.

Using the joint statistics and multiple detections not only of the same particle but also of different particles, Fig. 4 shows that a combined detection of one neutron and one photon, can be much more frequent than the doublet of one of them. Using the joint statistics and applying realistic detector efficiencies, one can use the analytical model to decide in a fast and relatively easy manner whether both types of particles should be detected, and if so, which combined factorial moments should be measured for best statistics.

4. Conclusions

Using proper application of symbolic computation, it has been demonstrated in previous work that high order terms of the number distribution can be derived and evaluated. This approach has now been used to incorporate also absorption and detection into the modelling of the probability distributions of neutrons and photons in a fissile sample.

The formal equivalence between the number distribution and factorial moments is also kept, meaning that factorial moments of very high orders can be easily calculated as limiting simplified evaluations of the formulae derived. The modified moments occurring in these expressions can all be readily calculated based only on nuclear data and characteristics such as sample mass, and detection efficiency.

The quantitative results show a good agreement with MCNP-PoliMi simulations. As can be expected, the inclusion of absorption does not have a significant effect on the neutrons; on the other hand, gamma photons are heavily attenuated. This has a considerable effect on the statistics of the emitted photons, whereas the neutron statistics is not much affected. Accounting for the process of detection by using detection efficiencies changes the statistics further. These studies are useful in giving indications as to what type of particle to focus on for assessing the sample with greatest accuracy.

Further, the model has been extended to be able to handle joint statistics of both neutrons and photons. Using joint moments adds to the diversity of the experimental methods, and can enhance the identification and detection of samples. The calculations again supply indications on how to design the measurement to get maximum efficiency. Using joint statistics might lead to shorter measuring times, or to get further data about the sample without having to use quadruplets.

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